

# Scattering of a damped inhomogeneous plane wave by a particle in a weakly absorbing medium

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**Abstract:** We use the volume integral equation formulation to consider frequency-domain electromagnetic scattering of a damped inhomogeneous plane wave by a particle immersed in an absorbing medium. We show that if absorption in the host medium is sufficiently weak and the particle size parameter is sufficiently small, then (i) the resulting formalism (including the far-field and radiative-transfer regimes) is largely the same as in the case of a nonabsorbing host medium, and (ii) one can bypass explicit use of sophisticated general solvers of the Maxwell equations applicable to inhomogeneous-wave illumination. These results offer dramatic simplifications for solving the scattering problem in a wide range of practical applications involving absorbing host media.

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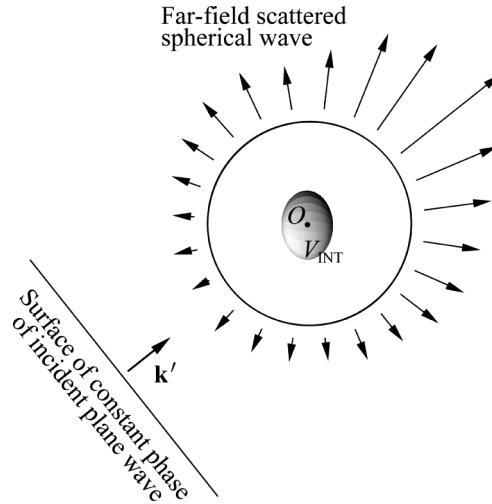
## 1. Introduction

Starting with the landmark volume by Stratton published in 1941 [1], electromagnetic scattering by particles embedded in an absorbing host medium has been the subject of great practical significance (see [2–20] and references therein). The majority of publications have dealt with the case of an impressed incident field modeled as a homogeneous (uniform) plane electromagnetic wave in which the planes of constant phase are parallel to the planes of constant amplitude. Yet in many applications the incident plane wave can be inhomogeneous. A typical example is the plane wave transmitted by a plane interface into an absorbing medium hosting particles [1,21,22]. The general theory describing this scenario becomes exceedingly involved and often impracticable. Yet it can be shown that if the host medium is weakly absorbing (which is the case in the majority of practical applications) then the theory can be simplified dramatically and essentially reduced to that developed for the case of a homogeneous incident wave. We demonstrate that in the rest of this paper.

## 2. Scattering problem

Our derivation is based on the general volume integral equation formulation (VIEF) of frequency-domain electromagnetic scattering for nonmagnetic materials (see [23–26] and references therein). We imply the monochromatic  $\exp(-i\omega t)$  dependence of all fields, where  $t$  is time,  $\omega$  is the angular frequency, and  $i = (-1)^{1/2}$ . The scattering problem is shown schematically in Fig. 1, in which a fixed finite object is embedded in an infinite medium that is assumed to be homogeneous, linear, isotropic, and in general absorbing. The object occupies an “interior” region  $V_{\text{INT}}$  filled with isotropic, linear, and possibly inhomogeneous materials and can include edges, corners, and intersecting internal interfaces [25]. (The assumption of isotropic materials is not essential; it is made for simplicity and can be relaxed.) Point  $O$  centered at the object serves as the

common origin of all position vectors. For further use we denote as  $R$  the radius of the smallest circumscribing sphere of the object centered at  $O$ .



**Fig. 1.** Scattering problem. The real part of the wave vector  $\mathbf{k}'$  is normal to surfaces of constant phase and generally is not parallel to the imaginary part of the wave vector  $\mathbf{k}''$ . In the far zone of the object, the scattered field becomes an outgoing transverse spherical wave.

Let us assume that the object is subjected to an impressed incident field  $\mathbf{E}^{\text{inc}}(\mathbf{r})$  in the form of a free-space solution of the macroscopic Maxwell equations. A fundamental result of the VIEF [26] is that the scattered field everywhere in space is given by

$$\mathbf{E}^{\text{sca}}(\mathbf{r}) = \int_{V_{\text{INT}}} d^3\mathbf{r}' \vec{G}(\mathbf{r}, \mathbf{r}') \cdot \int_{V_{\text{INT}}} d^3\mathbf{r}'' \vec{T}(\mathbf{r}', \mathbf{r}'') \cdot \mathbf{E}^{\text{inc}}(\mathbf{r}''), \quad (1)$$

where  $\vec{G}(\mathbf{r}, \mathbf{r}')$  is the free-space dyadic Green's function and  $\vec{T}(\mathbf{r}, \mathbf{r}')$  is the so-called transition dyadic. The latter is independent of the incident field, serves as a unique and universal “scattering ID” of the object in a given host medium, and can be found by solving the following integral equation of the Lippmann–Schwinger type:

$$\vec{T}(\mathbf{r}, \mathbf{r}') = U(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}')\vec{I} + U(\mathbf{r}) \int_{V_{\text{INT}}} d^3\mathbf{r}'' \vec{G}(\mathbf{r}, \mathbf{r}'') \cdot \vec{T}(\mathbf{r}'', \mathbf{r}'). \quad (2)$$

Here,

$$U(\mathbf{r}) \triangleq \begin{cases} 0, & \mathbf{r} \in \mathbb{R}^3 \setminus V_{\text{INT}}, \\ \omega^2 \varepsilon_2(\mathbf{r})\mu_0 - k_1^2, & \mathbf{r} \in V_{\text{INT}} \end{cases} \quad (3)$$

is the so-called potential function;  $\delta(\mathbf{r})$  is the three-dimensional delta function;  $\vec{I}$  is the identity (or unit) dyadic;  $\varepsilon_1$  and  $\varepsilon_2(\mathbf{r})$  are the complex permittivities of the host medium and the object, respectively;  $\mu_0$  is the magnetic permeability of a vacuum;

$$k_1 = k'_1 + ik''_1 \triangleq \omega\sqrt{\varepsilon_1\mu_0} \quad (4)$$

is the (generally complex) wave number in the host medium;  $\mathbb{R}^3$  is the entire three-dimensional space; and we express

$$\varepsilon_1 = \varepsilon'_1 + i\varepsilon''_1 = \varepsilon'_1(1 + i\tau), \quad (5)$$

where the parameter  $\tau = \varepsilon''_1/\varepsilon'_1 \geq 0$  quantifies the strength of absorption in the host medium. We have also assumed that  $\varepsilon'_1 > 0$  since otherwise the absorption cannot be small (see below).

If the incident and scattered electric fields are known everywhere in space then the corresponding magnetic fields  $\mathbf{H}^{\text{inc}}(\mathbf{r})$  and  $\mathbf{H}^{\text{sca}}(\mathbf{r})$  everywhere in space can be found from the frequency-domain Maxwell curl equations.

Let us assume that in general, the impressed incident field is a damped inhomogeneous plane electromagnetic wave given by [21,22]

$$\mathbf{E}^{\text{inc}}(\mathbf{r}) = \mathbf{E}_0 \exp(\mathbf{i}\mathbf{k} \cdot \mathbf{r}), \quad (6)$$

$$\mathbf{H}^{\text{inc}}(\mathbf{r}) = \mathbf{H}_0 \exp(\mathbf{i}\mathbf{k} \cdot \mathbf{r}), \quad (7)$$

where

$$\mathbf{k} = \mathbf{k}' + \mathbf{i}\mathbf{k}'' \quad (8)$$

is the complex wave vector in the host medium such that  $k' > 0$  and  $k'' \geq 0$ . Obviously, the planes of equal phase are normal to  $\mathbf{k}'$  (Fig. 1), while the planes of constant amplitude are normal to  $\mathbf{k}''$ . The amplitudes  $\mathbf{E}_0$  and  $\mathbf{H}_0$  specify the incident field at the origin  $O$  of the object-centered reference frame (Fig. 1).

The general properties of the plane wave are the following [1,21,22,24]:

$$\mathbf{k} \cdot \mathbf{E}_0 = \mathbf{k} \cdot \mathbf{H}_0 = 0, \quad (9)$$

$$\mathbf{H}_0 = (\omega\mu_0)^{-1} \mathbf{k} \times \mathbf{E}_0, \quad (10)$$

$$\mathbf{k} \cdot \mathbf{k} = k_1^2 = \omega^2 \varepsilon_1 \mu_0. \quad (11)$$

The dispersion Eq. (11) implies that

$$k'^2 - k''^2 = \omega^2 \varepsilon_1' \mu_0, \quad (12)$$

$$\mathbf{k}' \cdot \mathbf{k}'' = k' k'' \cos \zeta = \frac{1}{2} \omega^2 \varepsilon_1' \mu_0 \tau, \quad (13)$$

where  $\zeta$  is the angle between the vectors  $\mathbf{k}'$  and  $\mathbf{k}''$ . The non-negativity of the right-hand side of Eq. (13) implies that  $0 \leq \zeta \leq \pi/2$ . If  $\zeta = 0$  then  $\mathbf{k}' \parallel \mathbf{k}''$ , and the plane wave (6)–(8) is homogeneous. If  $\zeta = \pi/2$  then the host medium is nonabsorbing, but the wave is inhomogeneous (e.g., an evanescent one).

The Maxwell equations alone do not constrain the direction of  $\mathbf{k}''$  (and hence the angle  $\zeta$ ) for a given  $\mathbf{k}'$ , so it must be found from appropriate boundary conditions. For example, if the inhomogeneous wave is generated by refraction of a homogeneous wave in a nonabsorbing medium through a plane interface into an absorbing host then  $\mathbf{k}''$  is always normal to the interface [1,21,22].

Assuming that  $\zeta$  is known, it is easily verified that the solution of the system of Eqs. (12)–(13) is given by (cf. [21])

$$k' = \omega \sqrt{\varepsilon_1' \mu_0} \left\{ \frac{1}{2} \left[ \sqrt{1 + \left( \frac{\tau}{\cos \zeta} \right)^2} + 1 \right] \right\}^{1/2}, \quad (14)$$

$$k'' = \omega \sqrt{\varepsilon_1' \mu_0} \left\{ \frac{1}{2} \left[ \sqrt{1 + \left( \frac{\tau}{\cos \zeta} \right)^2} - 1 \right] \right\}^{1/2}. \quad (15)$$

These formulas conclude the generic outline of the scattering problem relevant to the case of an inhomogeneous plane incident wave.

### 3. Weakly inhomogeneous host medium

Let us further assume that the host medium is weakly absorbing (i.e.,  $\tau \ll 1$ ), and that  $\zeta$  is not too close to  $\pi/2$  so that

$$\tau/\cos \zeta \ll 1. \quad (16)$$

The actual requisite smallness of  $\tau/\cos \zeta$  depends on the specific circumstance, but values like  $10^{-2}$  or smaller could be mentioned as representing useful applications. Then

$$k' \approx \omega \sqrt{\epsilon'_1 \mu_0} \approx k'_1, \quad (17)$$

$$k'' \approx k' \frac{\tau}{2 \cos \zeta}, \quad k'' \ll k'. \quad (18)$$

Equations (17) and (18) are quite consequential. Indeed, we can conclude that to high accuracy,

$$\mathbf{k}' \perp \mathbf{E}_0, \quad \mathbf{k}' \perp \mathbf{H}_0, \quad \mathbf{H}_0 = (\omega \mu_0)^{-1} \mathbf{k}' \times \mathbf{E}_0. \quad (19)$$

In other words, the complex electric and magnetic field vectors are parallel to the planes of constant phase and are mutually perpendicular. It is easily seen that Eq. (19) implies that the instantaneous real electric and magnetic field vectors are normal to each other:

$$\mathcal{H}^{\text{inc}}(\mathbf{r}, t) \perp \mathcal{E}^{\text{inc}}(\mathbf{r}, t). \quad (20)$$

Furthermore,  $\mathbf{k}'$  defines the direction of the time-averaged Poynting vector:

$$\mathcal{S}^{\text{inc}}(\mathbf{r}) = \frac{1}{2} \text{Re}[\mathbf{E}^{\text{inc}}(\mathbf{r}) \times \mathbf{H}^{\text{inc}}(\mathbf{r})] = \frac{1}{2\omega\mu_0} \exp(-2\mathbf{k}'' \cdot \mathbf{r}) |\mathbf{E}_0|^2 \mathbf{k}'. \quad (21)$$

Notably, the relations (19) and (20) are characteristic traits of a (homogeneous) plane wave propagating in a nonabsorbing medium.

The scattered field in the far zone of the particle is an outgoing spherical wave [26]:

$$\mathbf{E}^{\text{sca}}(\mathbf{r}) = \mathbf{E}_1(\hat{\mathbf{r}}) \exp(ik_1 r), \quad (22)$$

$$\mathbf{H}^{\text{sca}}(\mathbf{r}) = \mathbf{H}_1(\hat{\mathbf{r}}) \exp(ik_1 r), \quad (23)$$

where  $\hat{\mathbf{r}} = \mathbf{r}/r$  is a unit vector in the scattering direction. This wave is always homogeneous in the sense of surfaces of constant amplitude coinciding with surfaces of constant phase. Again, it is straightforward to show (cf. Section 3.2 of [24]) that in the limit  $\tau \ll 1$ , we have

$$\hat{\mathbf{r}} \perp \mathbf{E}_1(\hat{\mathbf{r}}), \quad \hat{\mathbf{r}} \perp \mathbf{H}_1(\hat{\mathbf{r}}), \quad \mathbf{H}_1(\hat{\mathbf{r}}) = (\omega \mu_0)^{-1} k'_1 \hat{\mathbf{r}} \times \mathbf{E}_1, \quad (24)$$

$$\mathcal{H}^{\text{sca}}(\mathbf{r}, t) \perp \mathcal{E}^{\text{sca}}(\mathbf{r}, t), \quad (25)$$

$$\mathcal{S}^{\text{sca}}(\mathbf{r}) = \frac{1}{2} \text{Re}[\mathbf{E}^{\text{sca}}(\mathbf{r}) \times \mathbf{H}^{\text{sca}}(\mathbf{r})] = \frac{1}{2\omega\mu_0} \exp(-2k''_1 r) |\mathbf{E}_1(\hat{\mathbf{r}})|^2 k'_1 \hat{\mathbf{r}}. \quad (26)$$

By virtue of being the same as in the case of a nonabsorbing host medium, Eqs. (19), (20), (24), and (25) justify the introduction of the conventional transverse scattering dyadic  $\vec{A}(\hat{\mathbf{r}}, \hat{\mathbf{k}}')$  (and hence the  $2 \times 2$  amplitude scattering matrix) fully describing the scattered field in the far zone [26]:

$$\mathbf{E}^{\text{sca}}(\mathbf{r}) \underset{r \rightarrow \infty}{\approx} \frac{\exp(ik_1 r)}{r} \vec{A}(\hat{\mathbf{r}}, \hat{\mathbf{k}}') \cdot \mathbf{E}_0, \quad (27)$$

where

$$\begin{aligned} \vec{A}(\hat{\mathbf{r}}, \hat{\mathbf{k}}') &= \frac{1}{4\pi} (\vec{I} - \hat{\mathbf{r}} \otimes \hat{\mathbf{r}}) \cdot \int_{V_{\text{INT}}} d^3 \mathbf{r}' \exp(-ik_1 \hat{\mathbf{r}} \cdot \mathbf{r}') \\ &\times \int_{V_{\text{INT}}} d^3 \mathbf{r}'' \vec{T}(\mathbf{r}', \mathbf{r}'') \cdot (\vec{I} - \hat{\mathbf{k}}' \otimes \hat{\mathbf{k}}') \exp(i\mathbf{k}' \cdot \mathbf{r}'') \exp(-\mathbf{k}'' \cdot \mathbf{r}'') \end{aligned} \quad (28)$$

and  $\otimes$  is the dyadic product sign. Furthermore, Eqs. (19)–(21) and (24)–(26) allow one to give a meaningful definition of the Stokes parameters of the incident and scattered waves in terms of the

electric-field amplitudes  $\mathbf{E}_0$  and  $\mathbf{E}_1$  only [24,27]. This obviously paves the way to the standard (and self-contained) description of far-field scattering (Fig. 1) and radiative transfer in terms of the extinction and phase matrices [13,28].

Let us further assume that we can solve the above-formulated scattering problem rigorously and evaluate the scattered field everywhere in space for an arbitrary homogeneous plane incident wave specified by  $\mathbf{E}_0$ ,  $k'$ , and  $\hat{\mathbf{k}}' = \mathbf{k}'/k'$ , with  $\mathbf{k}'' \parallel \mathbf{k}'$  and  $k''$  being the non-negative solution of Eq. (12). For example, in the case of a spherically symmetric scattering object this can be done quite efficiently using the classical Lorenz–Mie theory [1,18,29]. This rigorous scattered field will be denoted as  $\mathbf{E}_{\text{hom}}^{\text{sca}}(\mathbf{r})$ . Using Eq. (1), this field can be expressed as

$$\mathbf{E}_{\text{hom}}^{\text{sca}}(\mathbf{r}) = \int_{V_{\text{INT}}} d^3\mathbf{r}' \vec{G}(\mathbf{r}, \mathbf{r}') \cdot \int_{V_{\text{INT}}} d^3\mathbf{r}'' \vec{T}(\mathbf{r}', \mathbf{r}'') \cdot \mathbf{E}_0 \exp(ik' \hat{\mathbf{k}}' \cdot \mathbf{r}'') \exp(-k'' \hat{\mathbf{k}}' \cdot \mathbf{r}''). \quad (29)$$

We also consider an arbitrary inhomogeneous plane incident wave (6)–(8) and denote the corresponding solution of the scattering problem as  $\mathbf{E}^{\text{sca}}(\mathbf{r})$ . Then

$$\mathbf{E}^{\text{sca}}(\mathbf{r}) = \int_{V_{\text{INT}}} d^3\mathbf{r}' \vec{G}(\mathbf{r}, \mathbf{r}') \cdot \int_{V_{\text{INT}}} d^3\mathbf{r}'' \vec{T}(\mathbf{r}', \mathbf{r}'') \cdot \mathbf{E}_0 \exp(ik' \hat{\mathbf{k}}' \cdot \mathbf{r}'') \exp(-k'' \hat{\mathbf{k}}'' \cdot \mathbf{r}''), \quad (30)$$

where  $\hat{\mathbf{k}}'' = \mathbf{k}''/k''$ . Owing to Eq. (19), we can choose a homogeneous wave with the same  $\mathbf{E}_0$  and  $\hat{\mathbf{k}}'$ , but generally different  $k'$ ,  $k''$ , and  $\hat{\mathbf{k}}''$ .

Finally, we assume that in either case the absorption size parameter of the scattering object is very small:

$$k''R \ll 1 \quad (31)$$

in both Eq. (29) and Eq. (30). For example, one may require that  $k''R \lesssim 0.01$ . Then for all  $\mathbf{r}'' \in V_{\text{INT}}$ ,  $\exp(-k'' \hat{\mathbf{k}}' \cdot \mathbf{r}'') \approx 1$  in Eq. (29) and  $\exp(-k'' \hat{\mathbf{k}}'' \cdot \mathbf{r}'') \approx 1$  in Eq. (30). Moreover, in these two equations  $k'$  can be replaced by  $k'_1$  owing to

$$(k' - k'_1)R = \frac{k''}{k' + k'_1}(k''R) \ll 1, \quad (32)$$

where we have used Eqs. (12), (18), and (31). As a consequence, we have to high precision (e.g., a few percent or better):

$$\mathbf{E}^{\text{sca}}(\mathbf{r}) \approx \mathbf{E}_{\text{hom}}^{\text{sca}}(\mathbf{r}) \quad (33)$$

and

$$\vec{A}(\hat{\mathbf{r}}, \hat{\mathbf{k}}') \approx \vec{A}_{\text{hom}}(\hat{\mathbf{r}}, \hat{\mathbf{k}}'), \quad (34)$$

where  $\vec{A}_{\text{hom}}(\hat{\mathbf{r}}, \hat{\mathbf{k}}')$  is the amplitude scattering matrix computed for the homogeneous plane incident wave. The same, of course, is true of the corresponding amplitude scattering matrices as well as extinction and phase matrices.

#### 4. Summary and discussion

In summary, we have shown that if absorption in the host medium is sufficiently weak, Eq. (16), then the traditional far-field and, by implication, radiative transfer formalisms remain virtually the same even if the incident field is an inhomogeneous plane wave. Furthermore, Eqs. (33) and (34) express the solution of the scattering problem for an inhomogeneous plane incident wave in terms of that for a homogeneous wave provided that the inequality (31) holds. As such, they represent a substantial simplification potentially applicable in many practical situations.

For example, the absorption in such ubiquitous bulk substances as water and water ice is weak enough at infrared wavelengths shorter than  $2 \mu\text{m}$  [30,31] to justify the inequality (16) in many

cases. If furthermore the scattering size parameter  $k'R$  of a particle embedded in water or ice is also sufficiently small then, depending on the actual smallness of  $k''/k'$ , the inequality (31) can become justified as well. This opens the possibility of using relatively simple and highly efficient solvers of the Maxwell equations developed for the case of a homogeneous plane incident wave (e.g., [18,29]) in place of more involved generalizations of the Lorenz–Mie theory [32,33] or universal solvers of the Maxwell equations such as, e.g., the VIEF-based discrete dipole approximation [34,35]. At longer wavelengths, absorption by water and water ice becomes so strong that it can be expected to annihilate many (if not all) measurable manifestations of light scattering (and especially of multiple scattering) in the first place [36].

The approximation (31) is in some sense analogous to the Rayleigh approximation based on the assumption that the incident field is nearly constant over the interior of the scattering object [37–41]. However, the range of practical applicability of the former can be much wider. Consider, for example, an object with the scattering size parameter  $k'R = 4\pi$  and the absorption size parameter  $k''R = 0.01$ . Obviously, the real and imaginary parts of the complex-exponential factor  $\exp(ik' \hat{\mathbf{k}}' \cdot \mathbf{r}'')$  in Eq. (30) can oscillate four times between  $-1$  and  $+1$  along the largest dimension of the scattering object, thereby rendering the Rayleigh approximation utterly inapplicable. Yet the real-valued absorption exponential factor  $\exp(-k'' \hat{\mathbf{k}}'' \cdot \mathbf{r}'')$  deviates by only  $\pm 0.01$  from its value unity at the origin (Fig. 1).

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